

# MASSIVE GRAVITY

(SPECULATIONS IN COSMOLOGY II)

MARK WYMAN

74TH COMPTON LECTURE SERIES

# SOME BUSINESS

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- No lecture next week! Happy Thanksgiving
- No lectures: Nov. 26th or Dec. 3rd
- Physics with a Bang! Dec. 3rd, 11am - 4pm
- Last lecture - with luncheon, Dec. 10th
- PS: I will discuss the superluminal neutrino news at the end



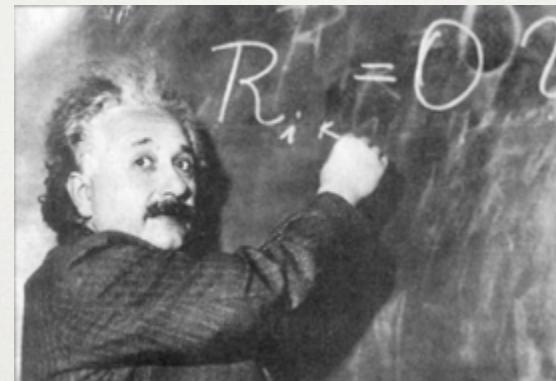
$m_g \neq 0?$

$$m_g^2 (h_{\mu\nu} h^{\mu\nu} - h^2)$$

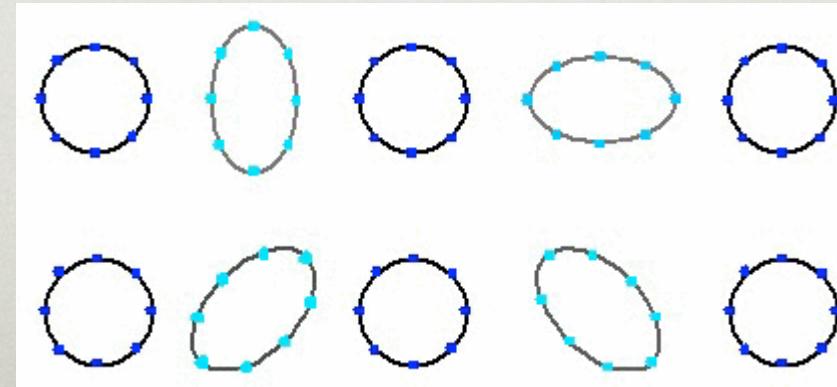
1939

# GENERAL RELATIVITY

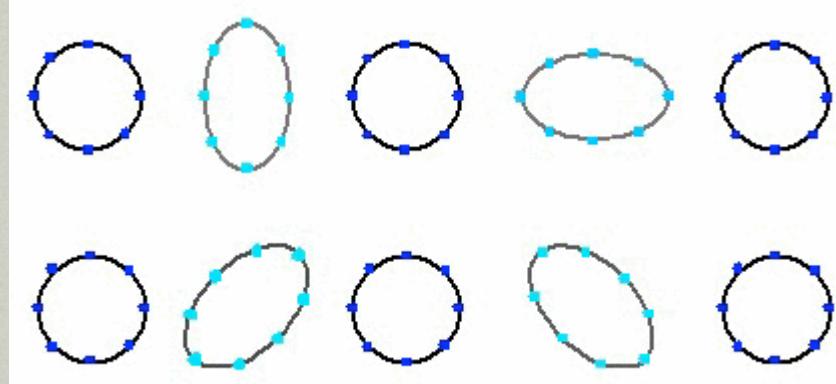
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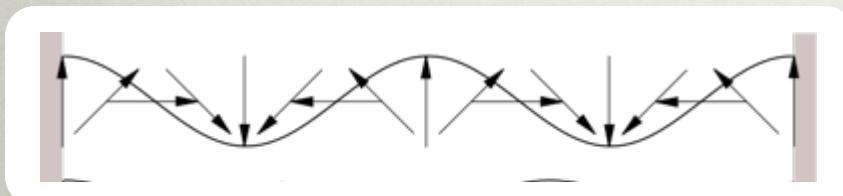
2 “tensor” fields



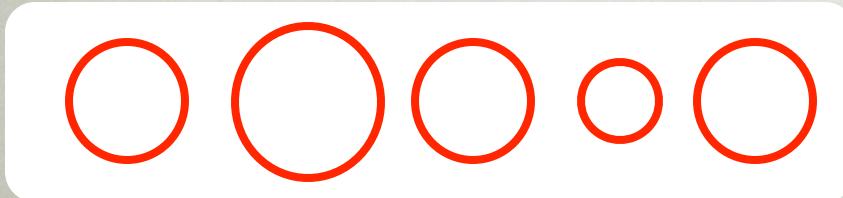
# MASSIVE GRAVITY



2 “tensor” fields



2 “vector” fields



1 scalar field

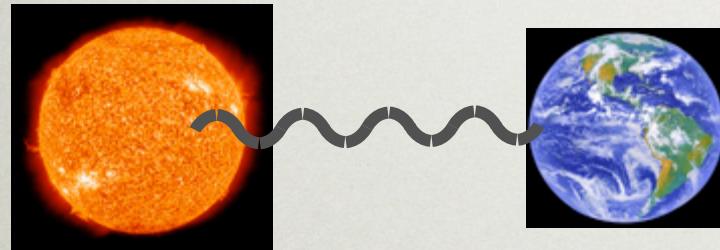


# VAN DAM, VELTMAN, ZAKHAROV DISCONTINUITY

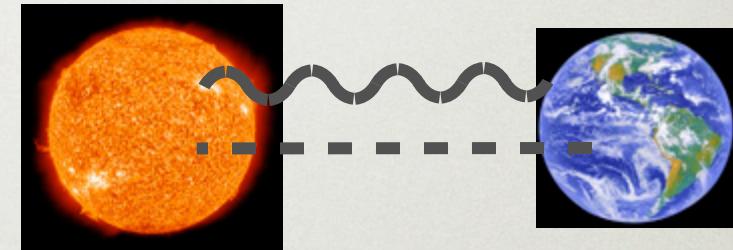
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1970

$$G \rightarrow \frac{4}{3}G$$



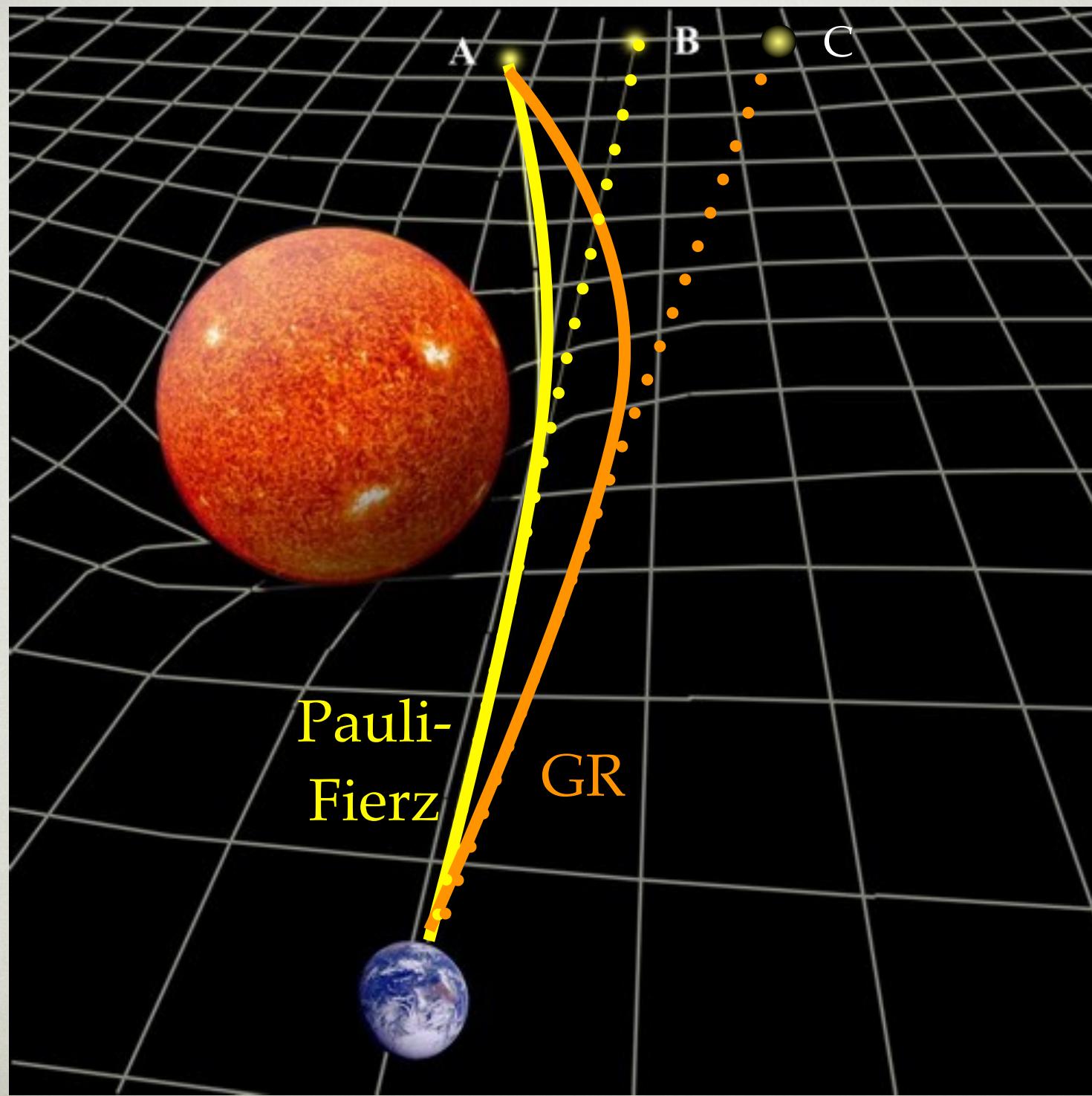
tensor graviton



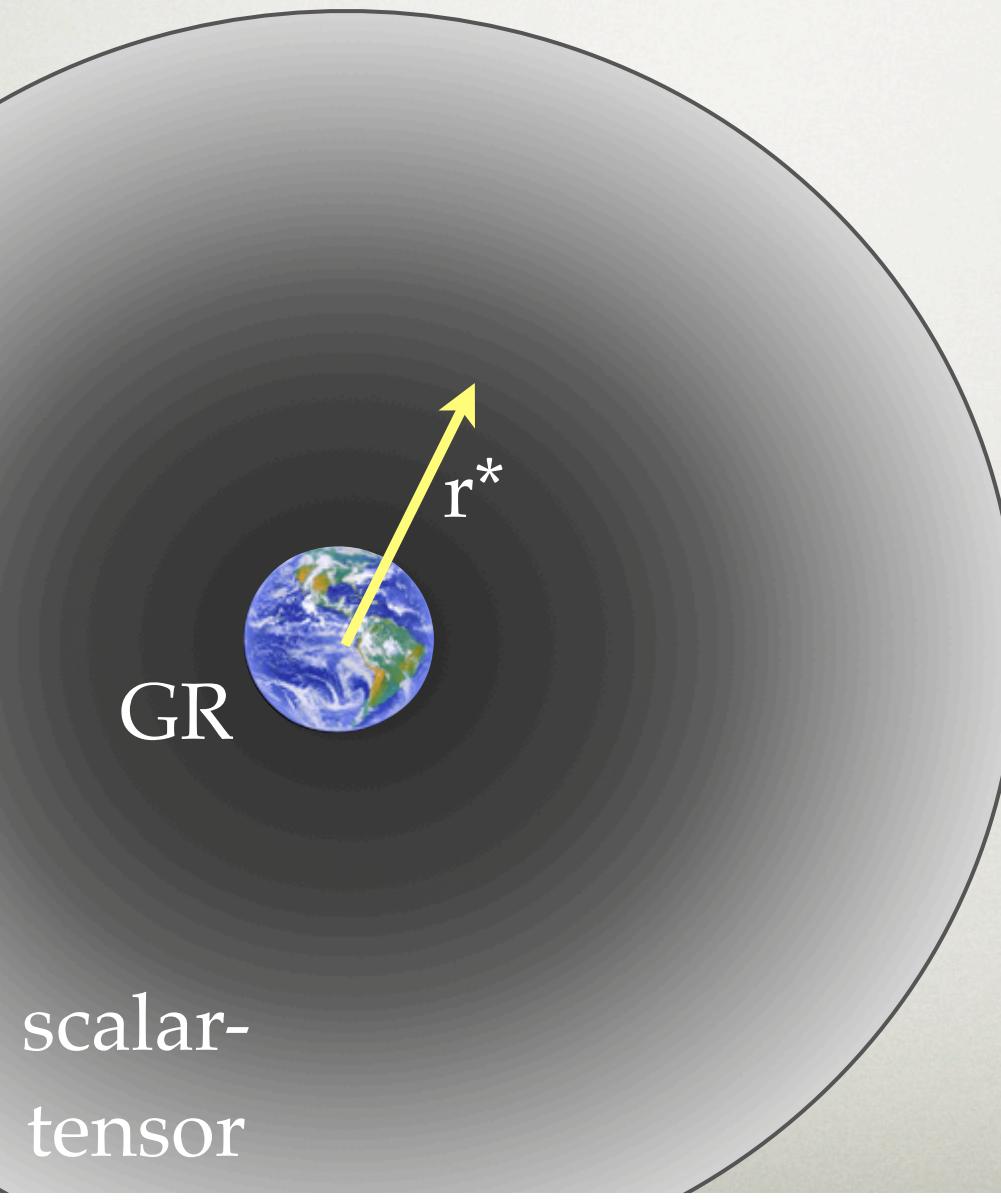
tensor graviton

+

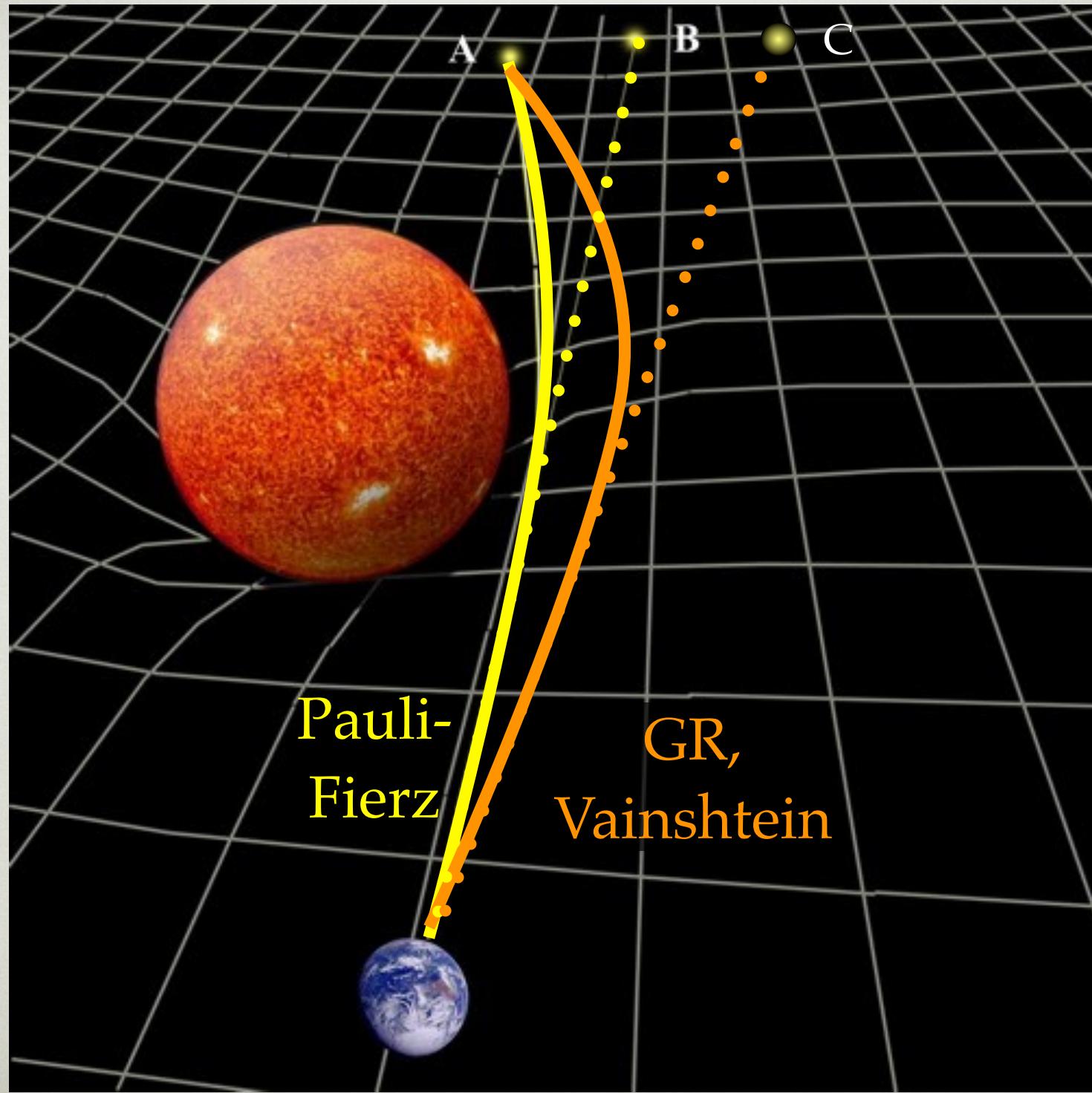
“scalar” graviton



# VAINSHTEIN EFFECT



1972

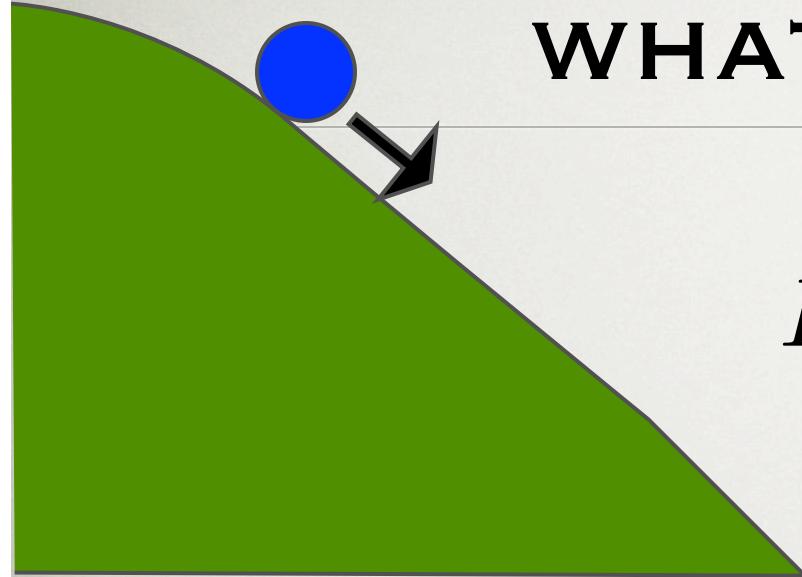


# **BOULWARE-DESER GHOST**

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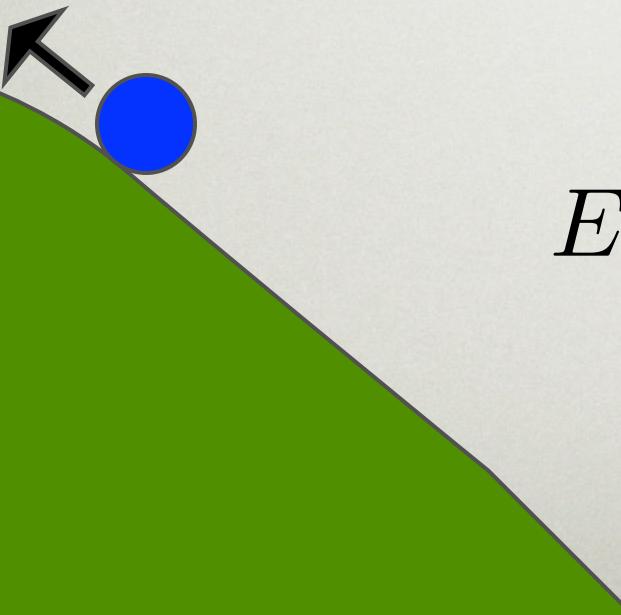
1972



# WHAT'S A GHOST?

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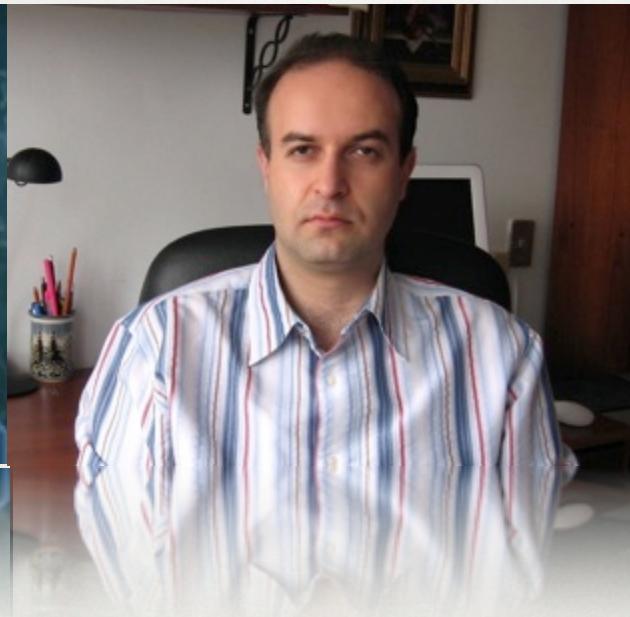
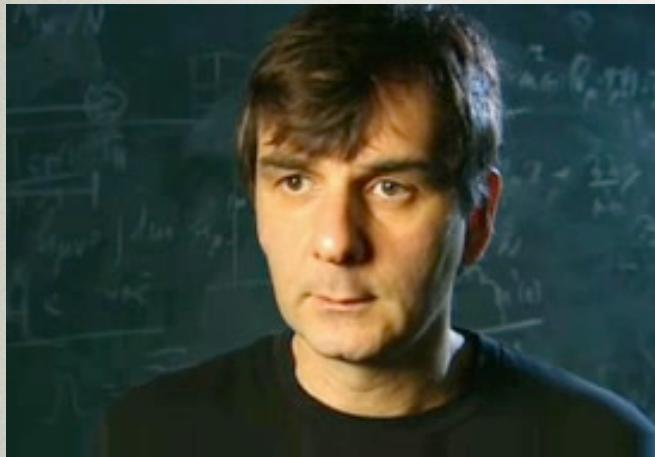
$$E = V + \frac{1}{2}mv^2$$



$$E = V - \frac{1}{2}mv^2$$



THE REVIVAL



D

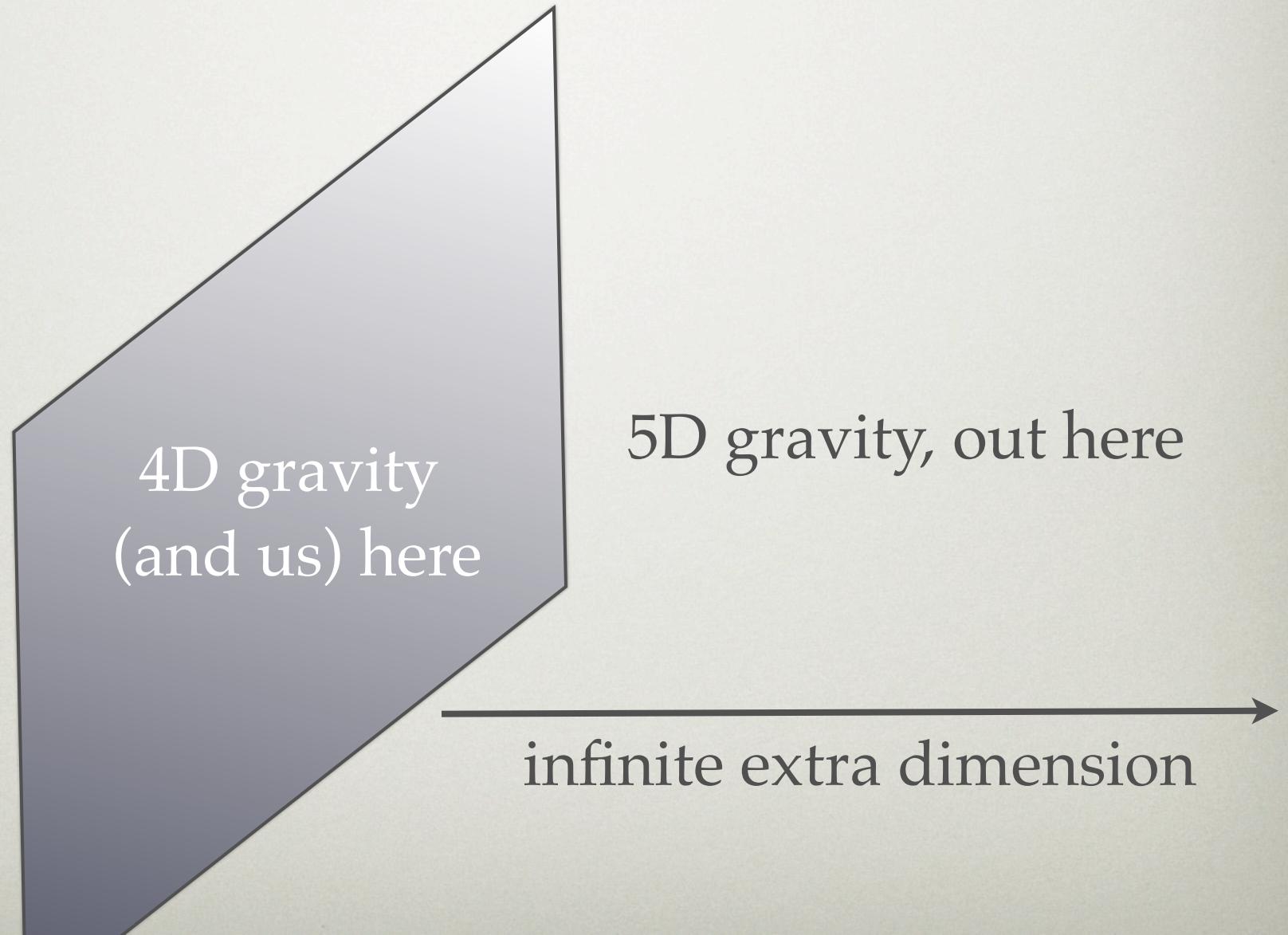
G

P

2001

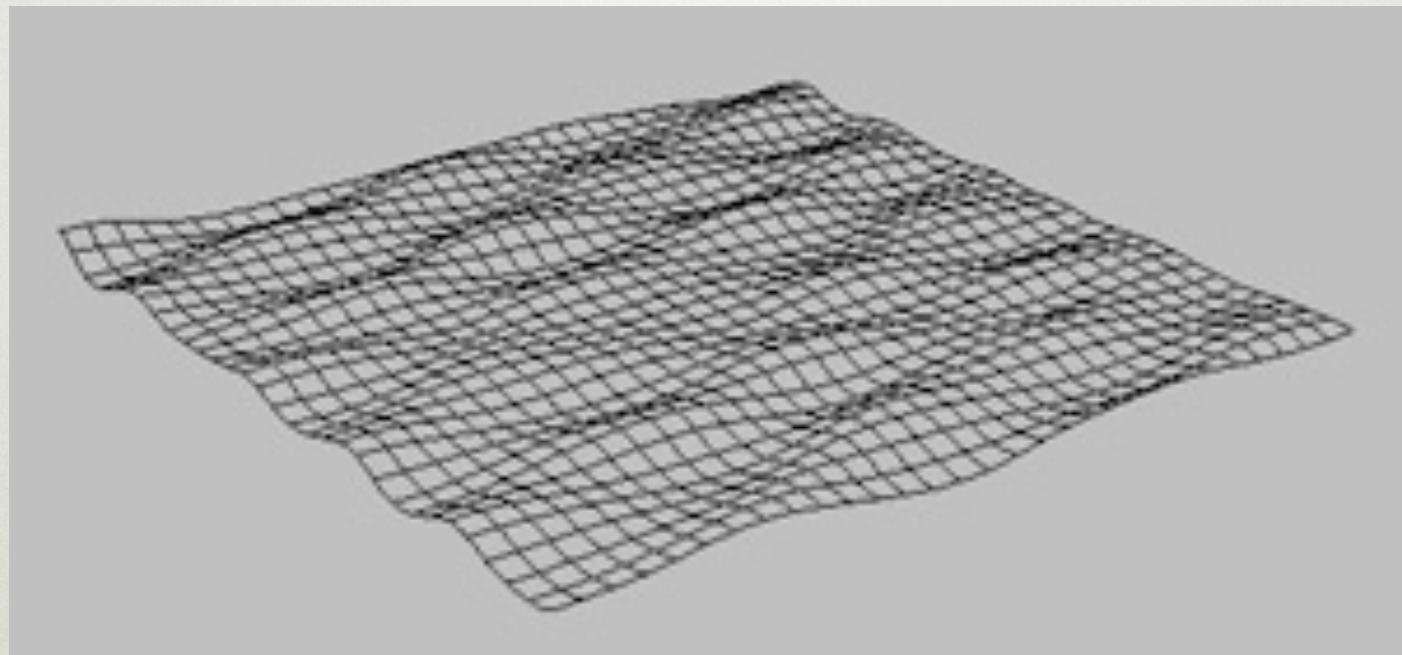
# THE DGP IDEA:

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# **“BRANE BENDING”**

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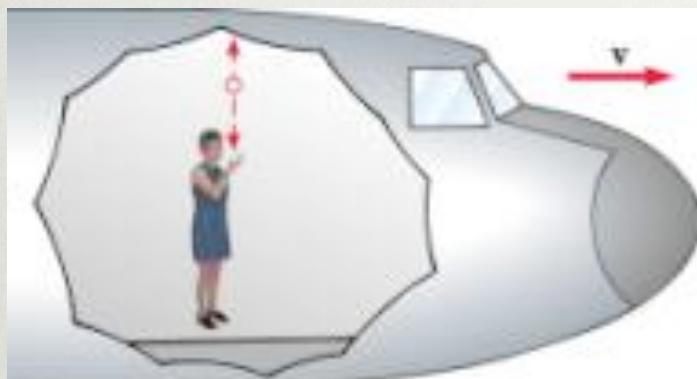


# THE GALILEAN SECRET

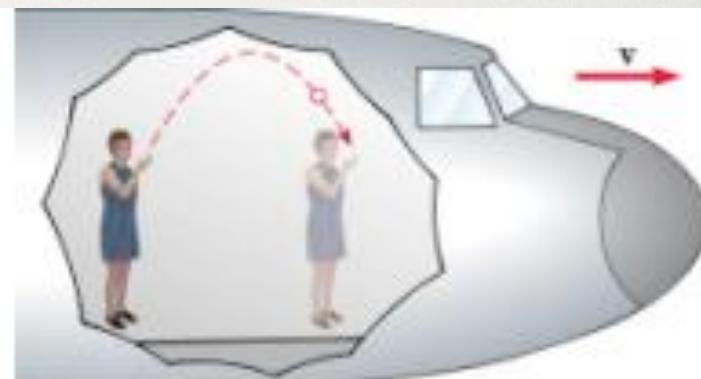
OF ONE

# GALILEAN SYMMETRY

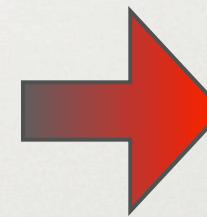
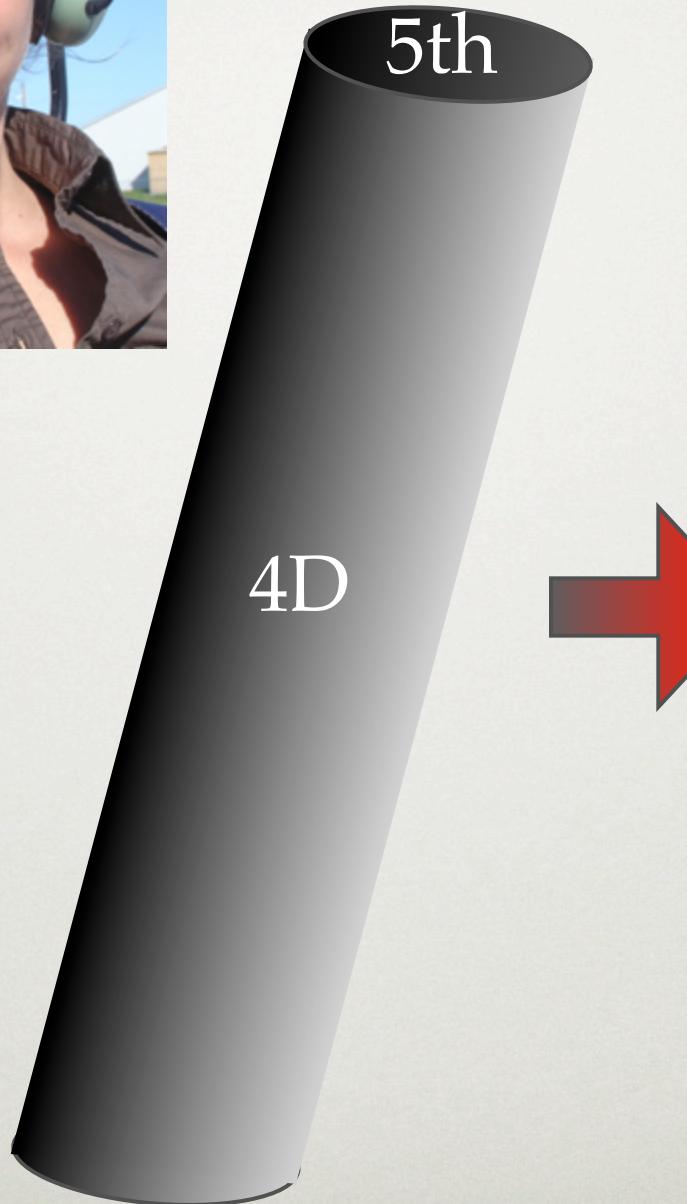
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(a)



(b)



graviton  
with mass!

2009



## purely 4D theory

**Formalism:** Define the tensor  $H_{\mu\nu}$  as the covariantization of the metric perturbation,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = H_{\mu\nu} + \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$ , where the four Stückelberg fields  $\phi^a$  transform as scalars, and  $\eta_{ab} = (-1, 1, 1, 1)$ , [3]. The helicity-0 mode  $\pi$  of the graviton can be extracted by expressing  $\phi^a = (x^a - \eta^{a\mu}\partial_\mu\pi)$ , such that

$$H_{\mu\nu} = h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\alpha\beta}\Pi_{\mu\alpha}\Pi_{\beta\nu}, \quad \Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi. \quad (1)$$

We may therefore define the following quantity

$$\mathcal{K}_\nu^\mu(g, H) = \delta_\nu^\mu - \sqrt{\delta_\nu^\mu - H_\nu^\mu} = -\sum_{n=1}^{\infty} d_n (H^n)_\nu^\mu, \quad (2)$$

$$\text{with } d_n = \frac{(2n)!}{(1-2n)(n!)^2 4^n}. \quad (3)$$

Here  $H_\nu^\mu = g^{\mu\alpha}H_{\alpha\nu}$ , and  $(H^n)_\nu^\mu = H_{\alpha_1}^\mu H_{\alpha_2}^{\alpha_1} \dots H_{\nu}^{\alpha_{n-1}}$  denotes the product of  $n$  tensors  $H_\beta^\alpha$ . Below, unless stated otherwise, all the contractions are made using the metric  $g_{\mu\nu}$ . The tensor  $\mathcal{K}_{\mu\nu} = g_{\mu\alpha}\mathcal{K}_\nu^\alpha$  is defined in such a way that

$$\mathcal{K}_{\mu\nu}(g, H)^\dagger = -A_{\mu\nu}. \quad (4)$$

no ghosts!

mid-2010

# CONTROVERSY ERUPTS!

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late-2010

# REALLY, NO GHOSTS!



$$S = -M_p^2 \int d^4x \sqrt{-g} \left[ R - 2m^2 \sum_{n=0}^3 \beta_n e_n(\sqrt{g^{-1}f}) \right], \quad (4)$$

where the  $\beta_n$  are given in terms of the  $\alpha_n$  of (3) as,

$$\begin{aligned} \beta_0 &= 6 - 4\alpha_3 + \alpha_4, & \beta_1 &= -3 + 3\alpha_3 - \alpha_4, \\ \beta_2 &= 1 - 2\alpha_3 + \alpha_4, & \beta_3 &= \alpha_3 - \alpha_4. \end{aligned} \quad (5)$$

The  $e_k(\mathbb{X})$  are elementary symmetric polynomials of the eigenvalues of  $\mathbb{X}$ . For a generic  $4 \times 4$  matrix they are given by,

$$\begin{aligned} e_0(\mathbb{X}) &= 1, \\ e_1(\mathbb{X}) &= [\mathbb{X}], \\ e_2(\mathbb{X}) &= \frac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]), \\ e_3(\mathbb{X}) &= \frac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]), \\ e_4(\mathbb{X}) &= \frac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]), \\ e_k(\mathbb{X}) &= 0 \text{ for } k > 4, \end{aligned} \quad (6)$$

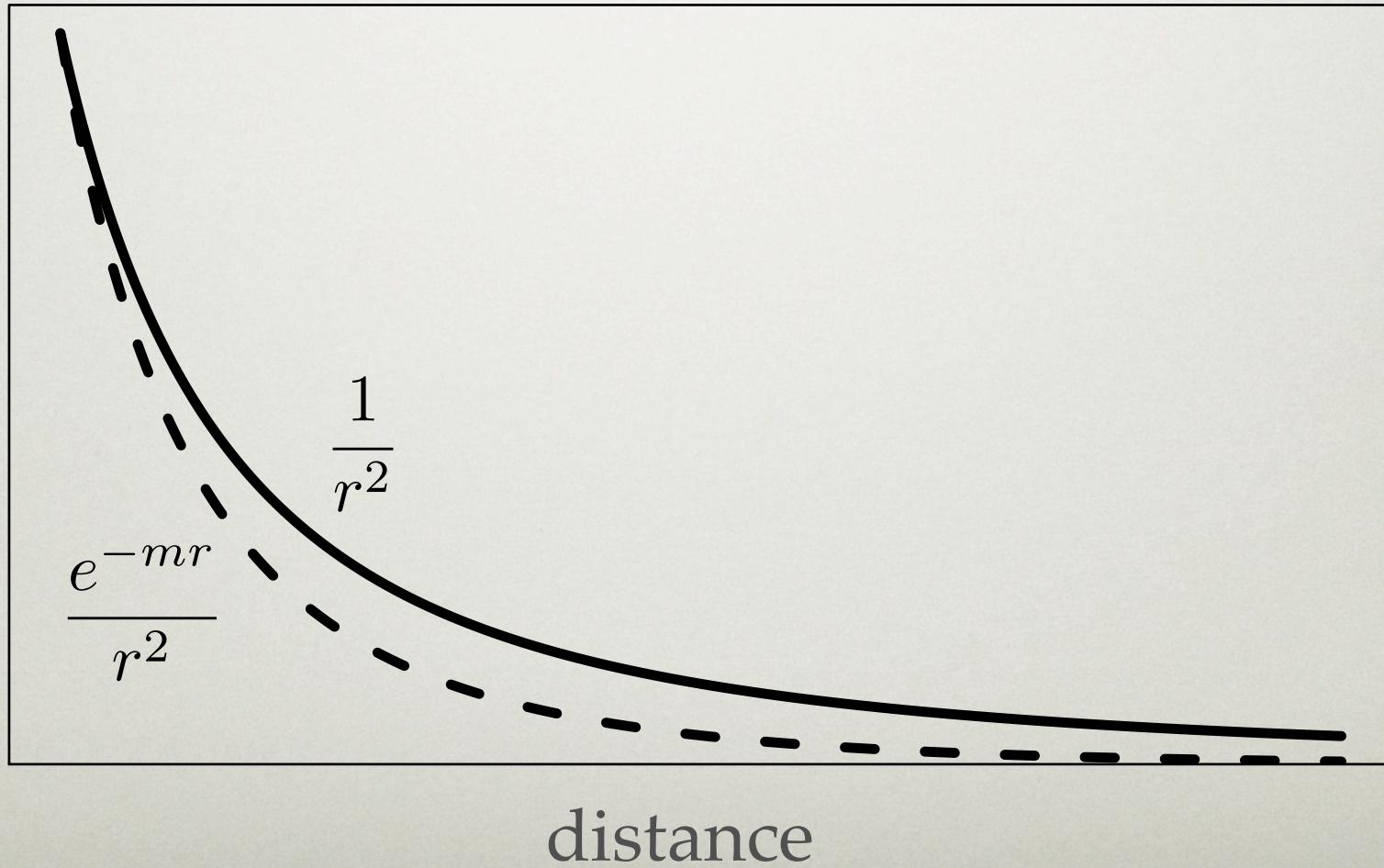
2011

# PHYSICAL CONSEQUENCES

# YUKAWA FORCE

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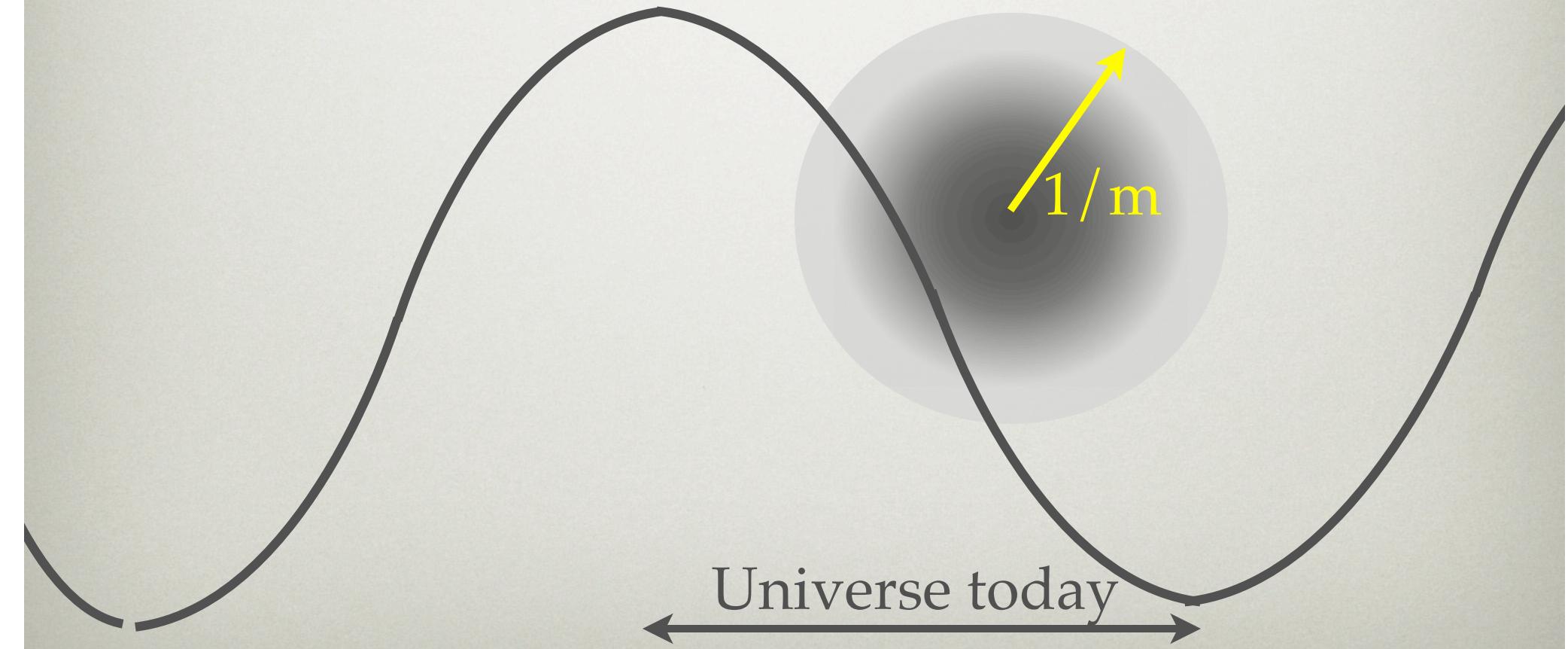
Force



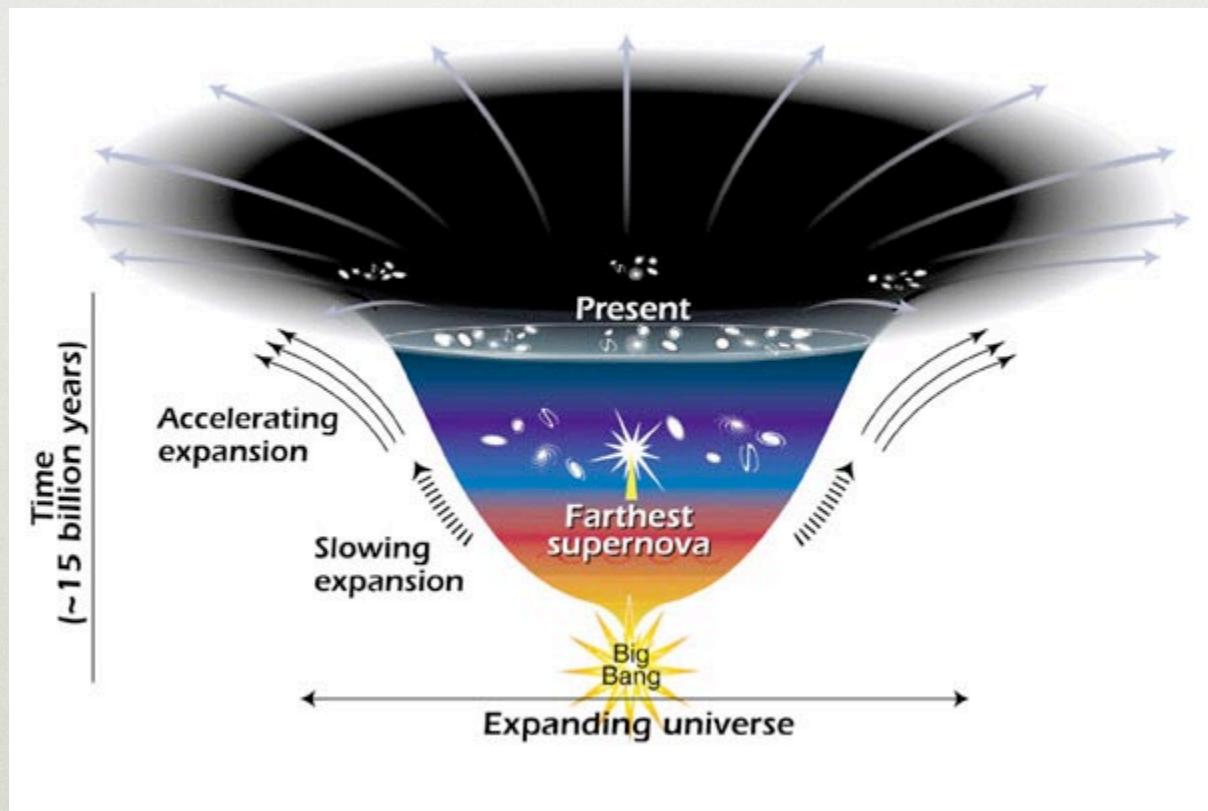
# “DEGRAVITATION” OF COSMOLOGICAL CONSTANT?

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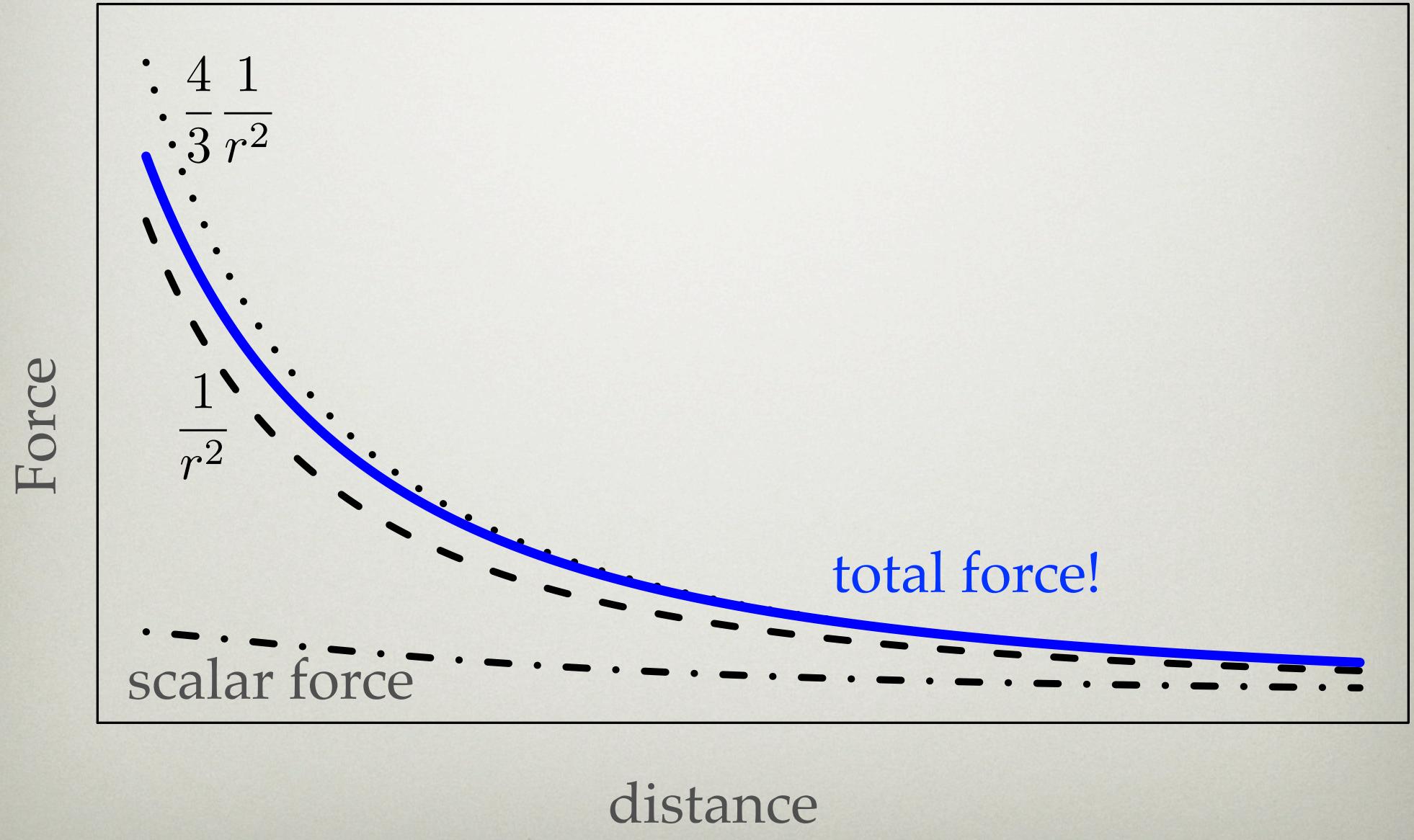
Yukawa = finite range force!



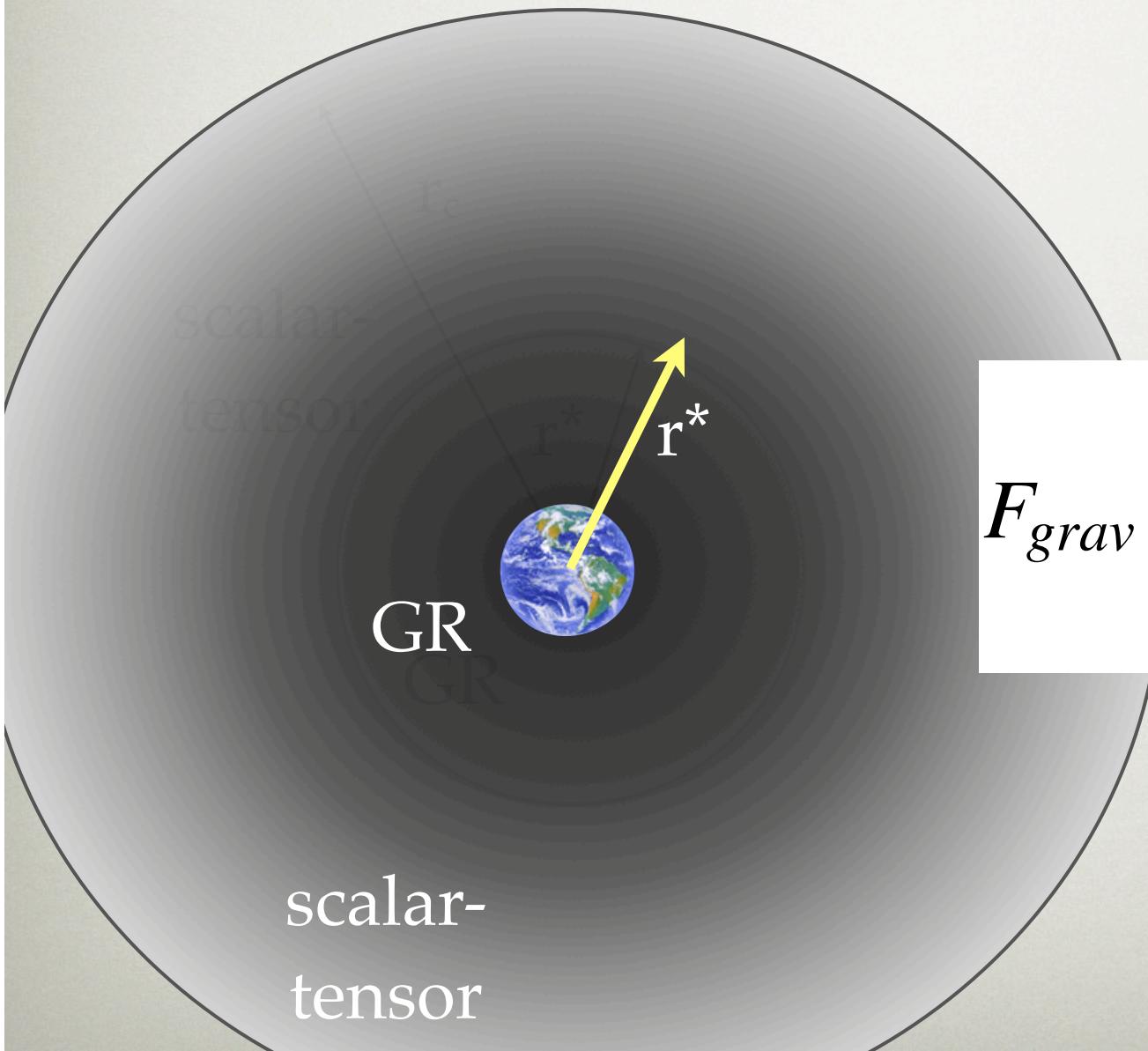
# OR: SELF ACCELERATION?



# EXTRA SCALAR FORCE



# $r^*$ / Vainshtein effect

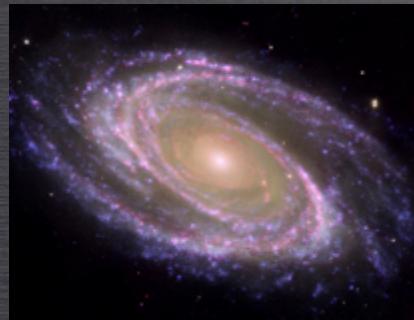


$$F_{grav} = \begin{cases} \frac{GM}{R^2} & R < r^* \\ \frac{4GM}{3R^2} & R > r^* \end{cases}$$

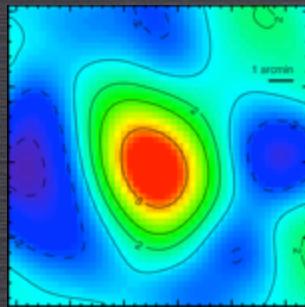
$$r^* = (r_{Sch} r_c^2)^{1/3}$$



$\sim \text{KPC} = 10^6 \text{ AU}$



$\sim \text{MPC} = 30 \text{ GALACTIC RADII}$

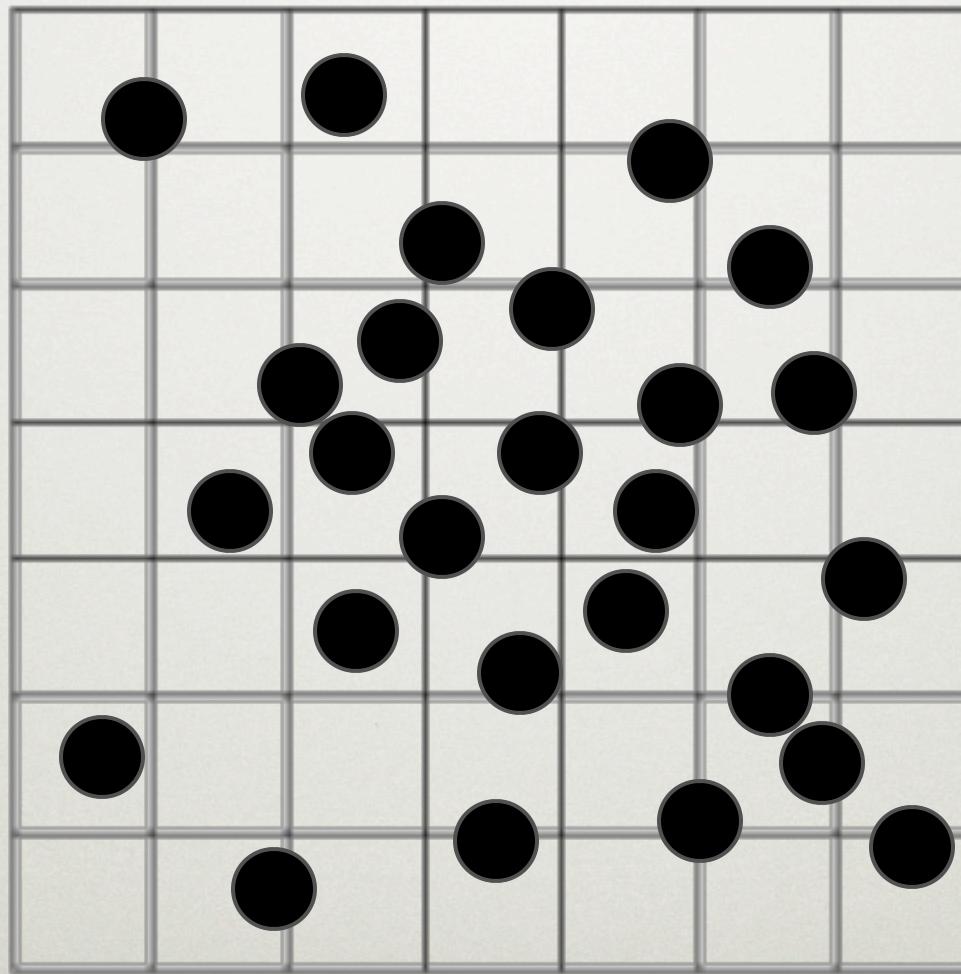


$\sim 10 \text{ MPC} = 10 \text{ VIRIAL RADII}$

# OBSERVATIONAL TESTS

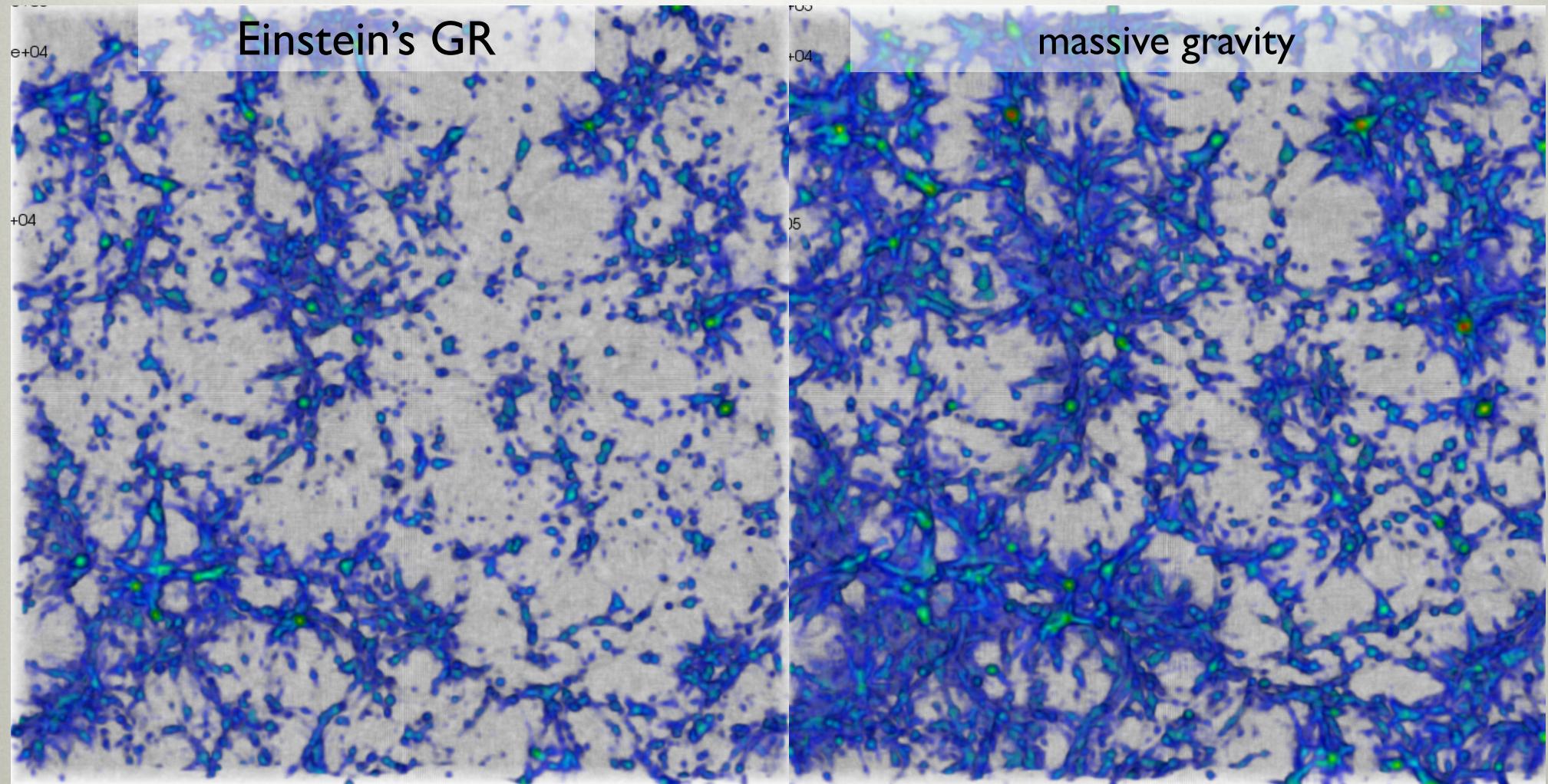
# N-BODY CODES

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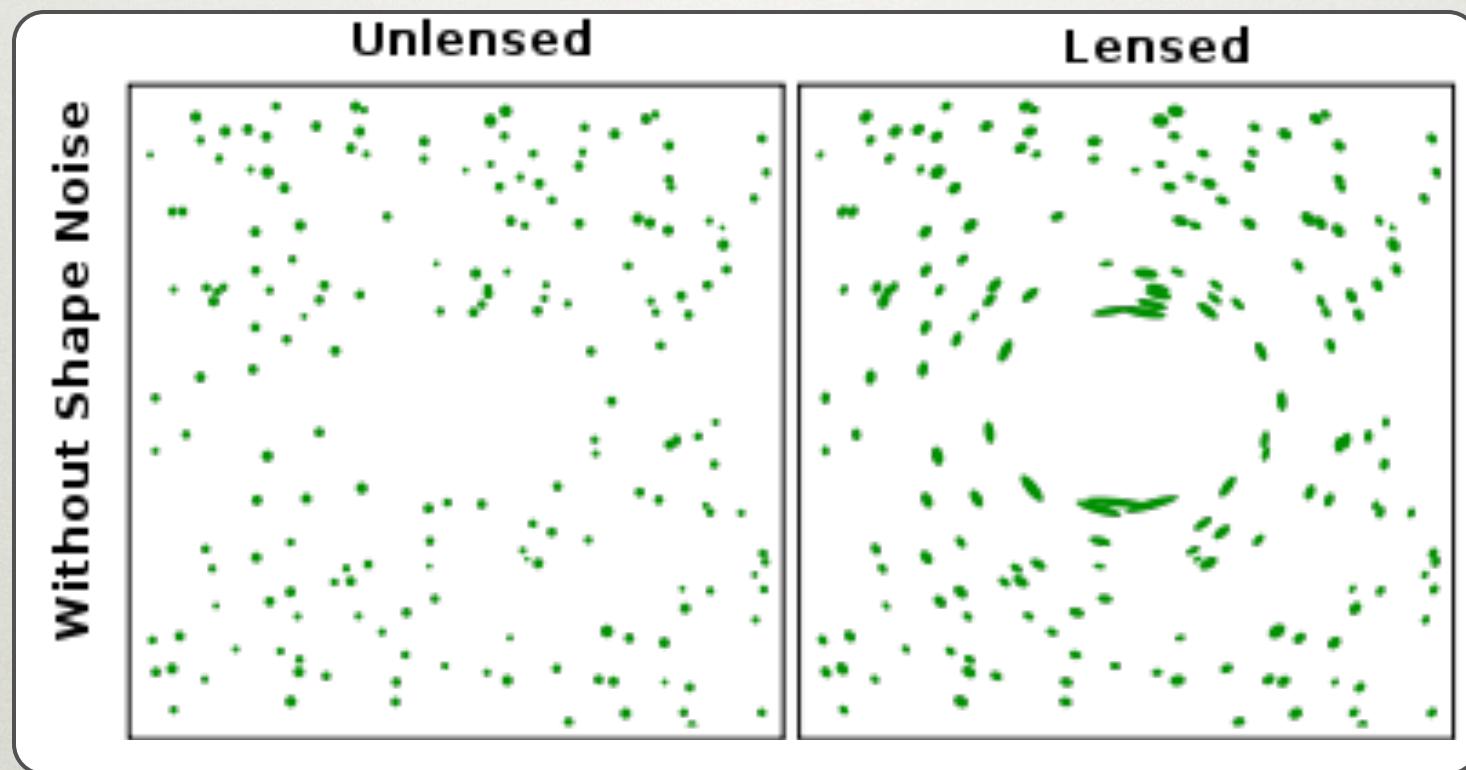
# SIMULATIONS OF THIS MODELS

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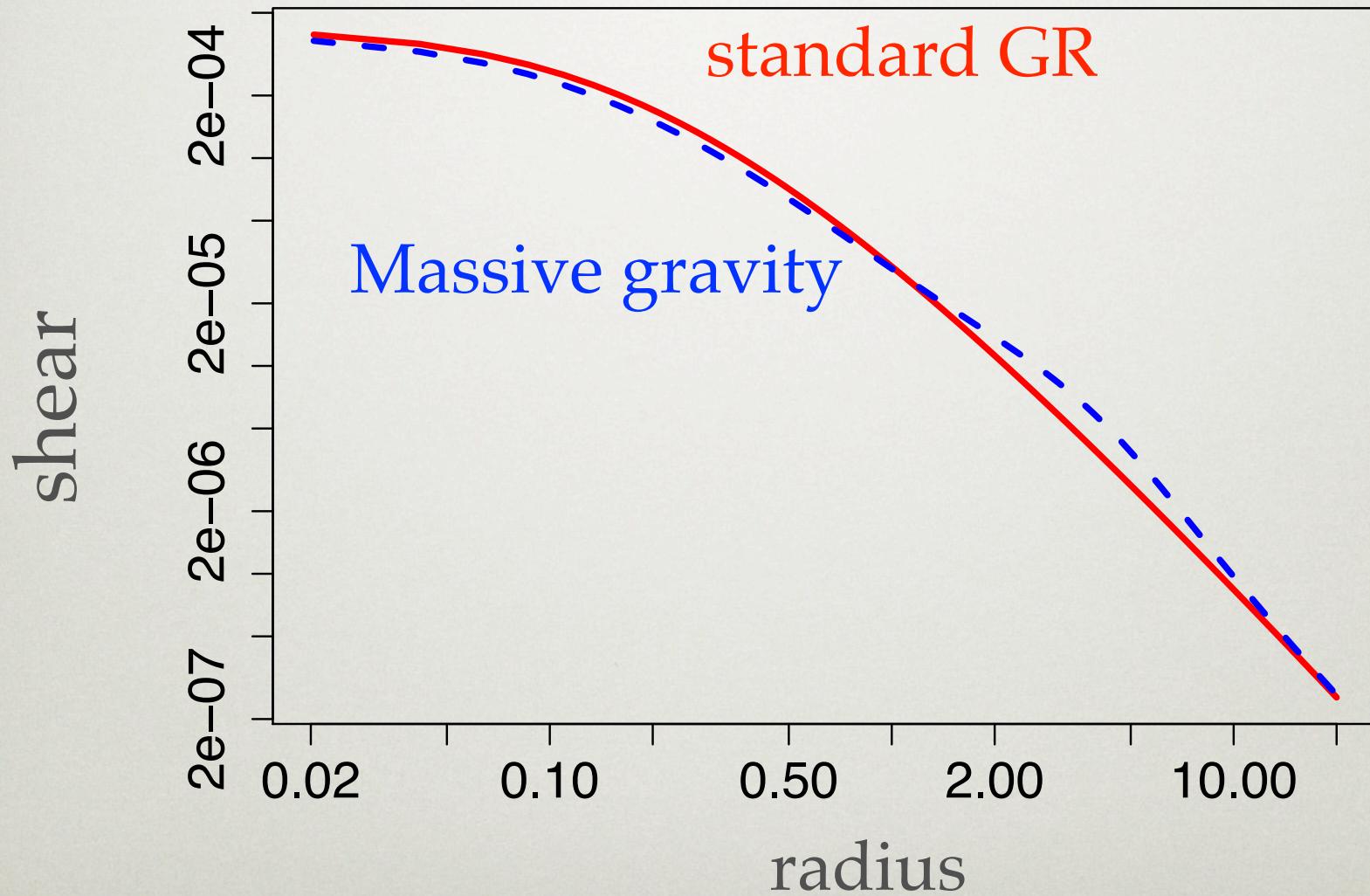
# WEAK LENSING

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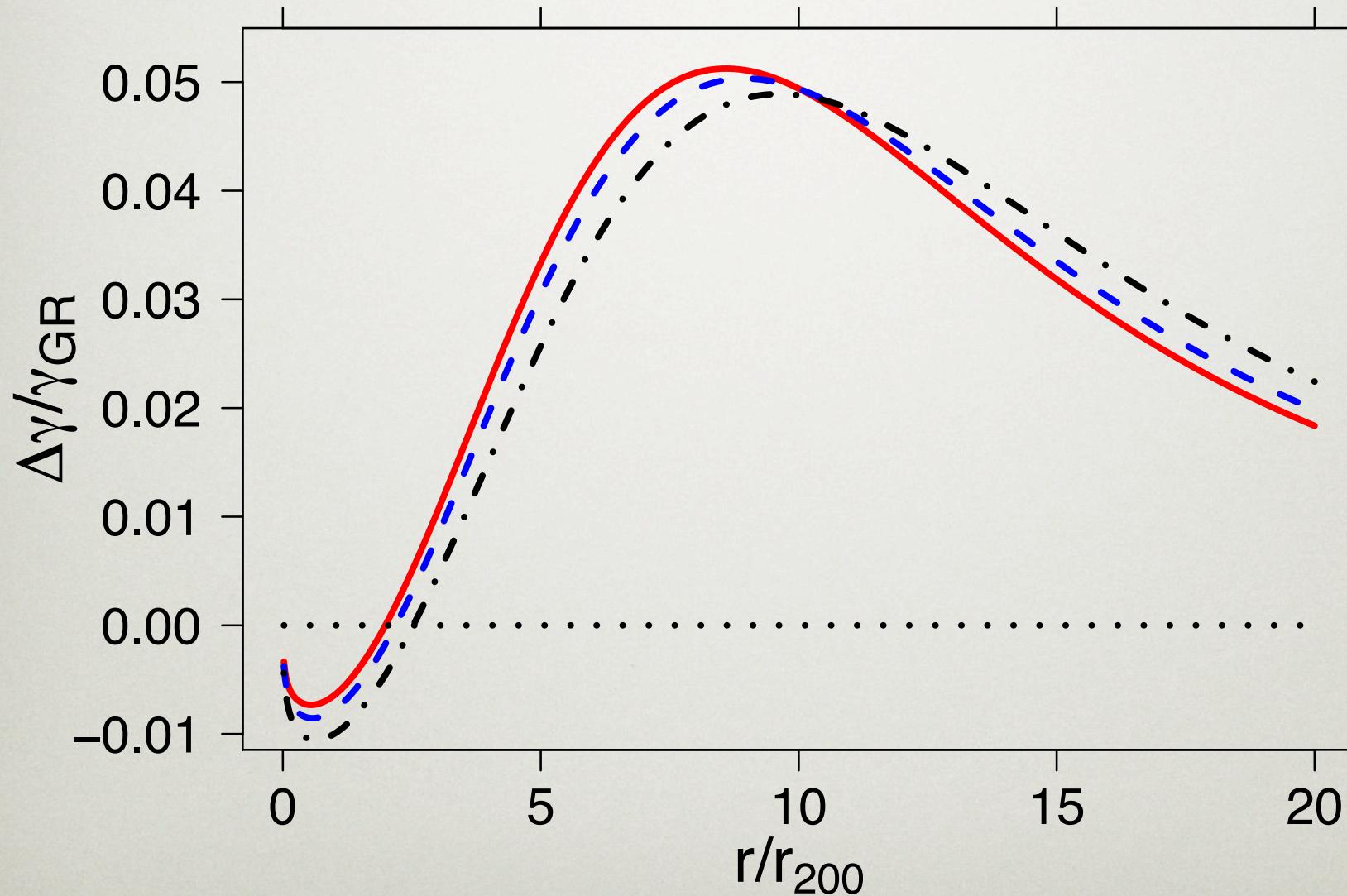


# PREDICTIONS

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## Modification to NFW tangential shear



# NEUTRINO UPDATE

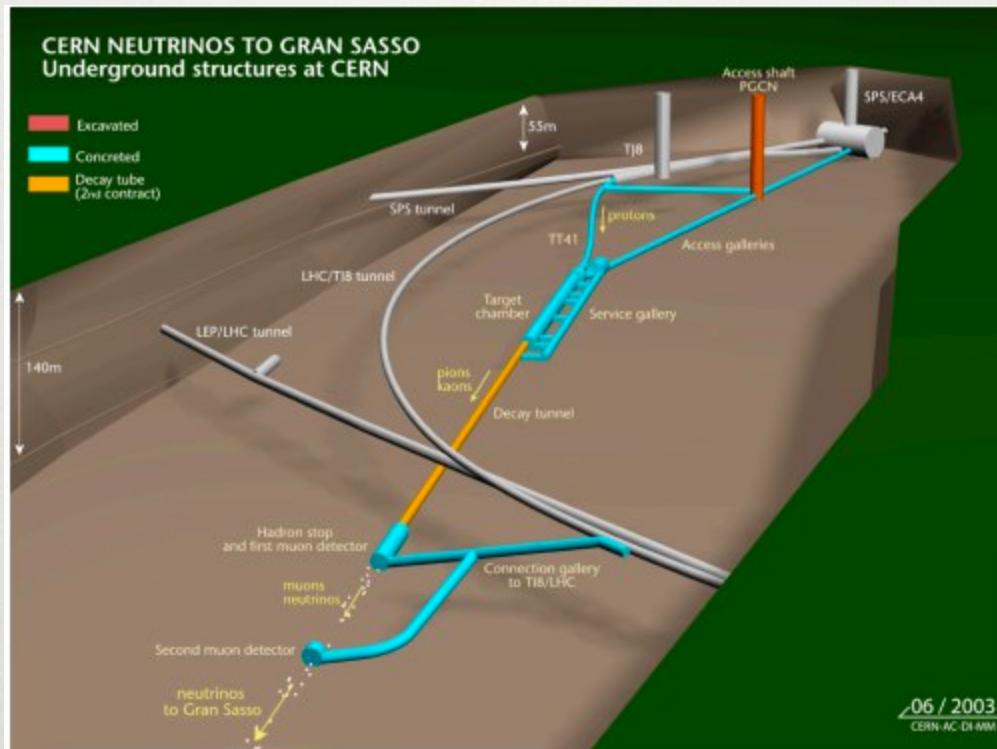


Fig. 1: Artistic view of the SPS/CNGS layout.

# NEUTRINO UPDATE

